2.36) a) $y_1 = t$, $y_2 = e^{-2t}$, $y_3 = \cos(3t)$

Entonces: $r_1 = 0$, $r_2 = 0$, $r_3 = -2$, $r_4 = 3i$, $r_5 = -3i$

$\rightarrow \boxed{y_H = k_1 + k_2 \cdot t + k_3 \cdot e^{-2t} + k_4 \cdot \operatorname{sen}(3t) + k_5 \cdot \cos(3t)}$

b) Propongo $y_P = (at+b) \cdot e^t$

Como antes vi que tiene 5 raíces y piden orden mínimo:

busco la expresión de la ec. que debe cumplir:

$r^2 \cdot (r+2) \cdot (r-3i) \cdot (r+3i) = 0$

$\rightarrow (r^3 + 2r^2)(r-3i)(r+3i) = 0$

$\rightarrow (r^3 + 2r^2)(r^2+9) = 0 \rightarrow r^5 + 9r^3 + 2r^4 + 18r^2 = 0 \rightarrow$

$\rightarrow y^{(5)} + 2y^{(4)} + 9y''' + 18y'' = 0$

entonces ahora uso y_P acá: $y^5 + 2y^4 + 9y''' + 18y'' = te^t$

$y_P' = ae^t + (at+b)e^t = e^t \cdot (a + at + b)$

$y_P'' = ae^t + a e^t + (at+b) e^t = e^t \cdot (2a + at + b)$

$y_P''' = ae^t + (2a + at + b)e^t = e^t \cdot (3a + at + b)$

$y_P^{(4)} = e^t \cdot (4a + at + b)$

$y_P^{(5)} = e^t \cdot (5a + at + b)$

b) $e^t \cdot (5a + at + b) + e^t \cdot (8a + 2at + 2b) + e^t \cdot (27a + 9at + 9b) + e^t \cdot (36a + 18at + 18b) = te^t$

$\rightarrow e^t \cdot (76a + 30at + 30b) = te^t$

$\cancel{\rightarrow 76a + 30at + 30b = t}$

$\rightarrow 30a = 1 \rightarrow a = 1/30$

$\rightarrow 76a + 30b = 0 \rightarrow \frac{76}{30} + 30b = 0 \rightarrow b = -\frac{\cancel{76}^{\cancel{38}^{19}}}{\cancel{900}_{\cancel{450}_{225}}} = -\frac{19}{225}$

Por lo tanto $\boxed{yp = \left(\frac{1}{30}t - \frac{19}{225}\right)e^t}$

c) Puedo proponer ~~...~~ y hacer mismo proced. que b)
$yp = at^3 + bt^2$

d) $L[y] = 5t + 8te^t$

Propongo $yp_1 = at^3 + bt^2$ (para $5t$) ①

$yp_2 = (ct + d)e^t$ (para $8te^t$) ②

① $yp_1' = 3at^2 + 2bt$ $\qquad yp_2' = e^t(c + ct + d)$

$yp_1'' = 6at + 2b$ $\qquad yp_2'' = e^t(2c + ct + d)$

$yp''' = 6a$ $\qquad yp_2''' = e^t(3c + ct + d)$

$yp^4 = 0$ $\qquad yp_2^4 = e^t(4c + ct + d)$

$yp^5 = 0$ $\qquad yp_2^5 = e^t(5c + ct + d)$

① $54a + 108a.t + 36b = 5t \rightarrow 108a = 5 \rightarrow \boxed{a = 5/108}$

$\rightarrow \cancel{54}^{1} \cdot \frac{5}{\cancel{108}_{2}} + 36b = 0 \rightarrow \boxed{b = -\frac{5}{72}}$

$\rightarrow \boxed{yp_1 = \frac{5}{108}t^3 - \frac{5}{72}t^2}$

② $e^t \cdot (5c+ct+d+8c+2ct+2d+27c+9ct+9d+36c+18ct+18d) = 8t\,e^t$

$\rightarrow e^t \cdot (76c+30ct+30d) = 8t\,e^t$

$30c = 8 \rightarrow \boxed{c = \frac{4}{15}}$

$76c + 30d = 0$

$\rightarrow 76 \cdot \frac{4}{15} + 30d = 0 \rightarrow 30d = -\frac{304}{15}$

$\rightarrow d = -\frac{304}{450} = \boxed{-\frac{152}{225}}$

$\rightarrow \boxed{y_{P2} = \left(\frac{4}{15} \cdot t - \frac{152}{225}\right) \cdot e^t}$

$y_P = y_{P1} + y_{P2} \rightarrow \boxed{y_P = \frac{5}{108} t^3 - \frac{5}{72} t^2 + \left(\frac{4}{15} t - \frac{152}{225}\right) e^t}$

la homogénea era $\boxed{y_H = k_1 + k_2 t + k_3 e^{-2t} + k_4 \cdot \sin(3t) + k_5 \cdot \cos(3t)}$

Por lo tanto:

$y_G = y_P + y_H$.

e)

$y_G(t) = k_1 + k_2 \cdot t + k_3 \cdot e^{-2t} + k_4 \cdot \operatorname{sen}(3t) + k_5 \cdot \cos(3t)$

Se que $y(0) = C_0$ (I), $y'(0) = C_1$ (II), $y''(0) = C_2$ (III), $y'''(0) = C_3$ (IV), $y^{(4)}(0) = C_4$ (V)

(I) $k_1 + k_3 + k_5 = C_0 \rightarrow k_1 = -k_3 - k_5 + C_0$

(II) $y'(t) = k_2 - 2k_3 e^{-2t} + 3k_4 \cos(3t) - 3k_5 \operatorname{sen}(3t)$

$k_2 - 2k_3 + 3k_4 = C_1 \rightarrow k_2 = 2k_3 - 3k_4 + C_1.$

(III) $y''(t) = 4k_3 e^{-2t} - 9k_4 \operatorname{sen}(3t) - 9k_5 \cos(3t)$

$4k_3 - 9k_5 = C_2 \rightarrow k_3 = \dfrac{C_2 + 9k_5}{4}$

(IV) $y'''(t) = -8k_3 e^{-2t} - 27k_4 \cos(3t) + 27k_5 \operatorname{sen}(3t)$

$-8k_3 - 27k_4 = C_3 \rightarrow k_3 = \dfrac{C_3 + 27k_4}{-8}$

(V) $y^{(4)}(t) = 16k_3 e^{-2t} + 81k_4 \operatorname{sen}(3t) + 81k_5 \cos(3t)$

$16k_3 + 81k_5 = C_4 \rightarrow k_5 = \dfrac{C_4 - 16k_3}{81}$

⋮

+ CUENTAS y salen los $k_1, \dots, k_5$.

F) Para q' $\lim_{t \to \infty} y(t) = 0$

entonces en

$y_G = k_1 + k_2 \cdot t + k_3 \cdot e^{-2t} + k_4 \cdot \operatorname{sen}(3t) + k_5 \cdot \cos(3t)$

$k_1 = k_2 = k_4 = k_5 = 0 \rightarrow y_G(0) = k_3.$

hacer lo mismo derivando y encontrando todas las cond. iniciales.